PHYS 328
IN - CLASS DISCUSSION 25 SEPT. 2012

We want to find the multiplicity of a two - state paramagnet of 10^{23} dipoles when half of the dipoles are pointing up and half pointing down. If we call the total number of dipoles N, then we know that:

$$\Omega = \binom{N}{N/2} = \frac{N!}{(N/2)!\,(N/2)!}$$

We write the factorials using Stirling' s approximation :

$$\Omega = \frac{N^N e^{-N} \sqrt{2\pi N}}{\left((N/2)^{N/2} e^{-N/2} \sqrt{2\pi (N/2)}\right)^2} = \frac{N^N e^{-N} \sqrt{2\pi N}}{(N/2)^N e^{-N} (\pi N)} = 2^N \sqrt{\frac{2}{\pi N}}$$

For such a large value of N, we can ignore the square root term since it is merely a large number, and regard the maximum multiplicity of this system as 2^N. We could have also reached this result by using Stirling's approximation in the form:

$$\ln N! = N \ln N - N$$

Using this version, our multiplicity becomes :

$$\Omega = \binom{N}{N/2} = \frac{N!}{(N/2)!\,(N/2)!} \Rightarrow \ln \Omega = \ln N! - 2 \ln (N/2)! =$$

$$N \ln N - N - 2\left((N/2) \ln (N/2) - (N/2)\right) = N \ln N - N \ln (N/2) = N \ln 2$$

Thus, if $\ln \Omega = N \ln 2$, then $\Omega = 2^N$

If the system changes 10^9 per second, there will be a total of $10^9/\text{s} * 10^{10}\text{yr} * \pi\, 10^7\text{s/yr} \approx \pi\, 10^{26}$ changes during the lifetime of the universe; this is approximately 2^{88} changes in the lifetime of the universe, clearly not remotely close to the total number of microstates of $2^{10^{23}}$.