WEAKLY INTERACTING EINSTEIN SOLIDS

In class, we followed the text' s analysis of Einstein' s solid. Let' s review the procedure, highlighting some of the details.

Our scenario is that we have two solids, A and B. A consists of 300 particles, B consists of 200 particles. The two solids are isolated from the outside environment and can share 100 units of energy.

Thus, the system can have 101 macrostates, ranging from 0 energy in solid A to all 100 units in solid A. The probability of finding A in any particular macrostate is :

$$\Omega_A (N_A, q_A) = \frac{(q_A + N_A - 1)!}{q_A! \, (N_A - 1)!}$$

Since we know that the total energy is conserved, the energy in B is simply the total energy minus the energy in A, we can write the probability of find B in a macrostate of energy $(q - q_A)$ is:

$$\Omega_B (N_B, q_A) = \frac{(q - q_A + N_B - 1)!}{(q - q_A)! \, (N_B - 1)!}$$

The total number of microstates available to the combined system of A and B corresponding to the macrostate q_A is:

$$\Omega_{total} (q_A) = \Omega_A (N_A, q_A) \, \Omega_B (N_B, q_A)$$

We can find the total number of microstates available for the entire system of 500 particles and 100 units of energy from :

```
In[267]:= totalstates = (100 + 500 - 1) ! / (100 ! (500 - 1) !) // N
```

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Out[267]= 9.26176 × 10^115
```

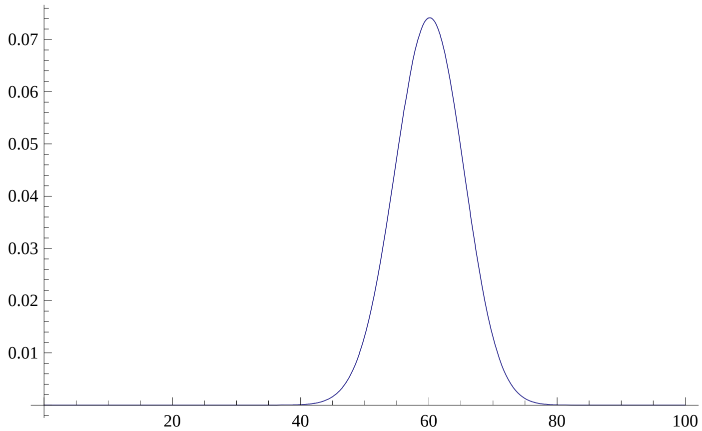
And we reproduce the value in the text. Now, the probability of obtaining any particular macrostate is simply

$$\text{probability of the macrostate } q_A =$$

$\Omega_{total} (q_A)$ / totalstates. Let' s write a short program to calculate this for us :

In[268]:=
```
Clear[na, nb, states, totalstates, statesa, statesb, prob]
na = 300; nb = 200; q = 100;
statesa[qa_] := (qa + na - 1) ! / (qa ! (na - 1) !)
statesb[qa_] := (q - qa + nb - 1) ! / ((q - qa) ! (nb - 1) !)
states[qa_] := statesa[qa] statesb[qa]
totalstates = Sum[states[qa], {qa, 0, q}];
prob[qa_] := states[qa] / totalstates
Plot[prob[qa], {qa, 0, q}, PlotRange → All]
```

Out[275]=

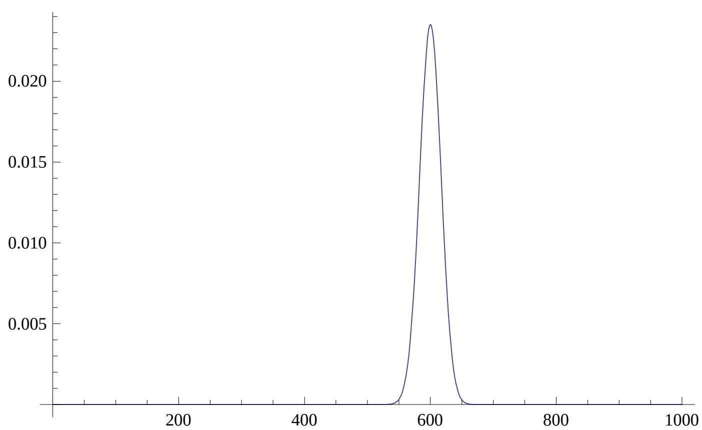


And we obtain a graph that reproduces our expected result, i.e., the most probable outcome occurs when 60 % of the energy is in solid A (which contains 60 % of the particles). Notice that in this program, I compute "totalstates" by actually summing the multiplicty of each macrostate. Let' s see if that computation yields the same number we had above :

In[277]:=
```
Print["the total number of microstates = ", totalstates // N]
```

the total number of microstates = 9.26176×10^{115}

What will the graph look like for larger values of N. Increasing all values by a factor of 10 (so that q = 1000 and na = 3000, nb = 2000) we get :



Use the code supplied to see if you can produce graphs for N = 50, 000 or even larger values of N. The point again is that as N increases, the probability distribution begins to spike around the value of the most likely event. Imagine continuing this procedure until we reach an Avogadro number of particles; the peak in that case would be indescribably narrow.