PHYS 328
FIRST HOUR EXAM

Fall 2013

This is a closed book, closed note exam. Use of calculators or other electronic devices is not permitted on this test. Do all your writing in your blue book, making sure your name is on each blue book you use. You may do questions in any order, but indicate clearly which question you are solving. All questions must be answered completely clearly showing the work you used to reach your solution. Answers with little to no justification/work will receive little to no credit. Please refer to the sheet of formulae at the end of this test.

1. Consider two tanks, A and B connected to each other by a valve which is closed at the outset. Tank A has twice the volume of tank B and both tanks are initially at 20^oC. If the pressure in tank A is initially 10^4Pa and the pressure in B is initially 10^5Pa, what will be the final temperature and pressure in each tank after the valve is opened? You may assume an ideal gas and that no heat is added or work is done on or by the gas. Make sure you explain both answers clearly and/or show your work. (25)

Appealing to the first Law of Thermodynamics, we can deduce that if no work is done on the gas and no heat is added, then the change in internal energy is zero. If the internal energy does not change, the temperature does not change, and the final temperature in the tanks will be the same as at the beginning.

The initial pressure in B is 10 times the initial pressure in A, and the volume of B is 1/2 of the volume of A. Therefore, we can use the ideal gas law to deduce that the initial number of particles in B is 5 times the number of particles in A. Quantitatively, we can show:

$$P_A V_A = N_A k T_A$$
$$P_B V_B = N_B k T_B$$

since $T_A = T_B$, $\dfrac{N_A}{N_B} = \dfrac{P_A V_A}{P_B V_B} = \dfrac{1}{10} \cdot 2 \Rightarrow N_A = 0.2\, N_B$

Once the valve is opened, the gas will reach equilibrium with a constant pressure throughout. The total number of particles is just the sum of particles in A and B, so :

$$N_{gas} = N_A + N_B = 0.2\, N_B + N_B = 1.2\, N_B$$

and the volume available to the gas is 3 times the volume of tank B. Therefore, applying the ideal gas law to the system after the valve is opened :

$$P_{gas}(3 V_B) = 1.2 N_B k T_B \Rightarrow P_{gas} = 0.4 \frac{N_B k T_B}{V_B} = 0.4 P_B = 4 \times 10^4 \text{ Pa}$$

2. Consider an Einstein solid with N oscillators and a total of q units of energy (N and q are both large) and it is appropriate to use the high temperature limit.

a) What can you deduce about the N/q ratio for this system? Justify (briefly) your answer. (5)

In the high temperature limit, there are many more units of energy than particles, so N >> q.

b) Using Stirling' s approximation as appropriate, find an expression for Ω (N, q) for this. (15)

Follow the treatment on pp. 63 - 64 of the text.

c) Find the entropy of the system. (10)

The entropy is given by S = k ln Ω, so that :

$$S = k \ln \Omega = k \ln\left[(e\, q / N)^N\right] = N k\, [\ln e + \ln(q/N)] = N k [1 + \ln(q/N)]$$

d) If the total energy can be expresses as U = ϵ q where ϵ is a constant, find an expression for the temperature of the system. (10)

The definition of entropy is :

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V} \Rightarrow \frac{1}{T} = \frac{\partial}{\partial U}[N k (1 + \ln(U/\epsilon N))] = N k \cdot \frac{1}{U/\epsilon N} \cdot \frac{1}{\epsilon N} = \frac{N k}{U}$$

$$\Rightarrow U = N k T$$

e) Derive an expression for the heat capacity of this system. (5)

$$C_V = \frac{\partial U}{\partial T} = N k$$

f) How many degrees of freedom does this system have? How do you determine that answer? (5)

By the equipartition theorem, the total thermal energy of a system is (f/2) N k T. Here, the coefficient of N k T is 1, so f = 2. For an oscillator in a solid, this is the result we expect.

3. Two identical bubbles of gas form at the bottom of a lake, then rise to the surface. Because the pressure at the surface is much lower than the pressure at the bottom, both bubbles expand as they rise. However, bubble A rises very quickly, so that no heat is exchanged between it and the water. Meanwhile, bubble B rises much more slowly so that is always remains in thermal equilibrium with the water (which we assume to be constant throughout the lake). Which of the two bubbles is larger by the time they reach the surface? Explain your reasoning fully. (25)

Since the two bubbles are identical, they have the same number of molecules. They both end at the surface, so have the same pressure (in equilibrium with atmospheric pressure), so the ideal gas law tells us that V = N k T/P, or that the volume of the bubble will depend on the temperature at the surface. Both bubbles expand and thus do work on the outside environment. But bubble B must be absorbing energy from the environment to maintain a constant temperature; bubble A rises adiabatically and does not absorb heat from the lake, so must experience a net loss in internal energy and thus has a lower temperature at the surface. Since B is warmer at the surface than A, B must be larger.

FORMULAE AND RESULTS

$$PV = nRT$$

$$PV = NkT$$

$$nR = Nk$$

$$N = nN_A$$

$$U = Nf \cdot \frac{1}{2} kT$$

$$\Delta U = Q + W$$

$$W = \mathbf{F} \cdot d\mathbf{r}$$

$$W = -\int_{V_i}^{V_f} P(V)\, dV$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$C_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P$$

$$\binom{n}{m} = \frac{n!}{m!\,(n-m)!}$$

$$\Omega(N, q) = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$N! = N^N e^{-N} \sqrt{2\pi N}$$

$$\ln N! = N \ln N - N$$

$$\Omega_N = \frac{1}{N!} \frac{V^N}{h^{3N}} \frac{\pi^{3N/2}}{(3N/2)!} \left(\sqrt{2mU}\right)^{3N}$$

$$\Omega(U, V, N) = f(N)\, V^N U^{3N/2}$$

$$S = k \ln \Omega$$

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + 5/2\right]$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V}$$

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$\ln(1+x) \approx 1 + x \quad |x| << 1$$