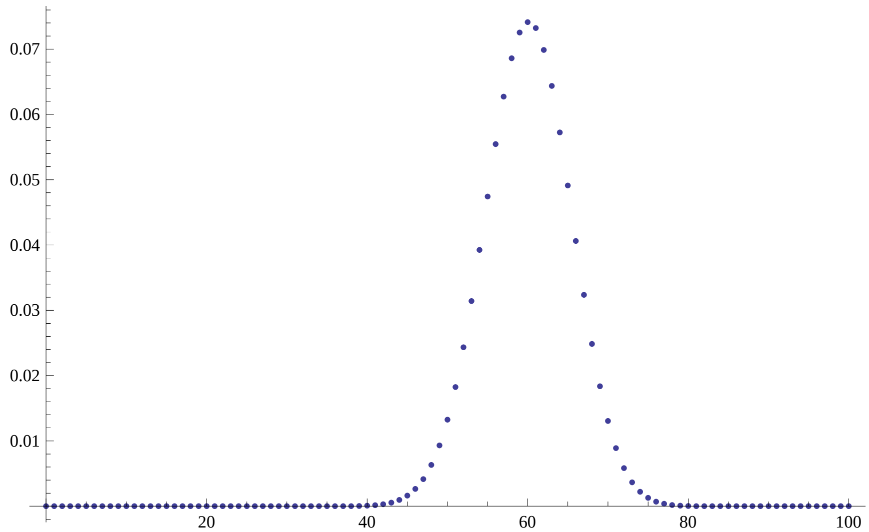
SHAPRNESS OF THE PROBABILITY DISTRIBUTION

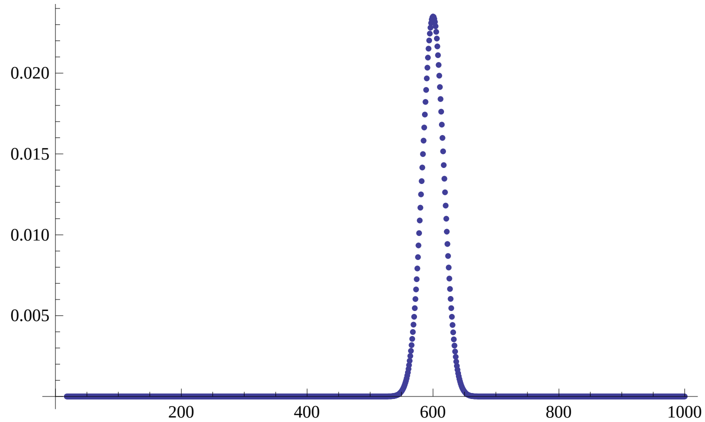
We have learned that computing the total number of available microstates is an important element in understanding thermodynamics and statistical mechanics. To understand the outcomes of interacting systems, we need to be able to determine which macrostates are the most probable results of the interaction.

Consider the situation presented in the text, system A with 300 particles interacts with system B of 200 particles sharing a total of 100 units of energy. As the text demonstrates, there are $9.27 * 10^{115}$ microstates available to the interacting system. We can compute the probability of each of the 101 possible resulting macrostates and produce the following graph:



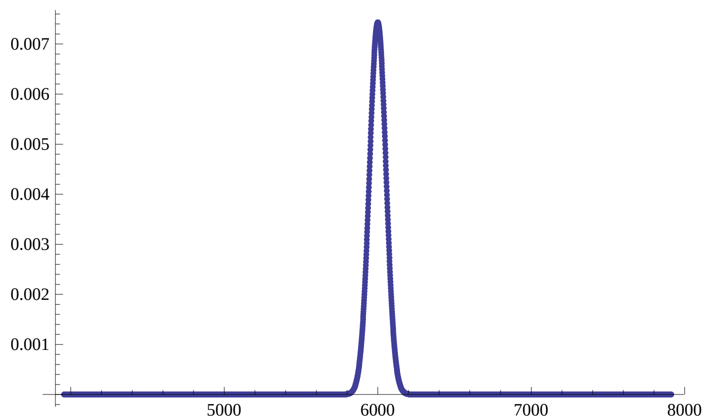
This graphs shows the probability of finding a particular macrostate as a function of the energy stored in system A. Notice that the maximum probability occurs for $q_a = 60$, and that the value of this maximum probability is just greater than 0.07.

Now, suppose we ask, what would this probability distribution look like if we increased all the parameters by a factor of 10? In other words, what is the distribution of the 1001 possible macrostates if now system A has 3000 particles, system B has 2000 particles, and the interacting system has a total of 1000 units of energy? Computing these probabilities, we obtain:



Notice that now the maximum probability occurs at q_a = 600 as you might have expected. Let's consider these two graphs and ask what if any similarities exist between them? What differences are there?

What do you think a graph of even more particles would look like? Below is the graph for the case where system A has 30,000 particles, system B has 20,000 particles, and the total energy is 10,000 units of energy. (This took a while for *Mathematica* to produce):



One of the things you might have noticed is that the peak of the probability distribution becomes narrower as the number of particles increases, indicating that the only likely resulting macrostates are those in the vicinity of the maximum probability.

How can we be sure that the narrowing of the peak is real and not just an artifact of the scale of the axes? Think of some quantitative measurements that we could make to determine whether the distribution is in fact narrowing.

Finally, while the differences between these three graphs demonstrate the behavior of larger systems, we need also to keep in mind that any real system we consider will have ***many*** more particles than the 50,000 considered in our largest case here. Section 2.4 of the text will teach us the mathematical tools needed to approximate such large systems, since you would wait a long time for any computer to produce these curves for a system of Avogardro's number of particles.