All homework assignments this semester must be submitted at the beginning of class on the due date. When Mathematica is required or allowed, include the Mathematica output with your homework. All problems must include clear and complete solutions to receive full credit. For this assignment, all calculus operations should be done by hand.

1. This question explores the relative cost of human vs. fossil fuel energy. A human being hard at work will expend approximately 100 W of power (if you use an exercise bike or treadmill that displays power, see how hard you have to work to expend 100 W). A small compact car traveling at highway speeds will expend approximately 100 kW. Use available data (such as the federal rate for reimbursing mileage) to determine how much money is required to operate a car for an hour. Now, suppose that the energy is provided by work study students pushing the car. How many students would you need to move the car at the same speed? Using the current U.S. minimum wage, how much would it cost to operate that car for an hour? Show all work and state explicitly all assumptions you make and values you use in your calculations.

2. In our study of thermodynamics, we will frequently compute partial derivatives of functions of several variables. This question illustrates the importance of being mindful which variable (s) is being held constant. Consider two functions, \( w = x y \) and \( x = y z \).
   a) Write \( w \) purely in terms of \( x \) and \( z \), and then purely in terms of \( y \) and \( z \).
   b) Compute the partial derivatives:
      \[
      \left( \frac{\partial w}{\partial x} \right)_y \text{ and } \left( \frac{\partial w}{\partial x} \right)_z
      \]
      where the subscript indicates the variable that is held constant. Show that these two partial derivatives are not equal.

3. On a clear autumn day in the midlatitudes, it is possible for the evening low to reach 30°F and the afternoon high to reach 60°F. Is it meaningful to say that it was twice as hot at the afternoon high than at the evening low? If so, explain why; if not, explain why not.

4. Determine the number of molecules in a cubic meter of air at the surface of the Earth at room temperature and 1 atm of pressure.

5. The number density of the atmosphere decreases exponentially with height according to:
   \[
   n(z) = n_0 e^{-z/H}
   \]
where \( n(z) \) is the number density at any height \( z \) above the surface of the Earth, \( n_o \) is the number density at the surface of the earth, and \( H \) is a constant called the scale height of the atmosphere. For the earth, the scale height is 8 km.

Find the total number of molecules in a 1 m\(^2\) column reaching from the surface of the Earth to the “top” of the atmosphere, several hundred kilometers above the surface of the Earth. Assuming the Earth is a sphere of radius \( R \) (\( R = 6400 \) km), estimate the total number of molecules in the Earth's atmosphere.