PHYS 328
HOMEWORK #2
Solutions

1. Using the more realistic temperature profile, the barometric equation becomes :

$$dP = -\frac{\mu m g}{k(T_0 - \alpha z)} P\, dz$$

separating variables gives us :

$$\frac{dP}{P} = -\frac{\mu m g}{k} \frac{dz}{T_0 - \alpha z}$$

Integrate both sides :

$$\ln P = \frac{\mu m g}{\alpha k} \ln(T_0 - \alpha z) + C = \ln(T_0 - \alpha z)^{(\mu m g/\alpha k)} + C$$

where C is a constant and we have made use of the properties of logs. Exponentiating both sides we obtain :

$$P(z) = A(T_0 - \alpha z)^{(\mu m g/\alpha k)}$$

where A is another constant.

---

2. a) The derivation in the text shows that for a single particle in a box of cross - sectional area A, the pressure contributed by the collision of a single particle is :

$$P = -\frac{m\left(\frac{\Delta v_x}{\Delta t}\right)}{A}$$

For N particles, we expect that the pressure will be :

$$P = -\frac{N m\left(\frac{\Delta v_x}{\Delta t}\right)}{A}$$

Solving for N and recalling that if the collisions with the container are elastic, $\Delta v_x = -2\, v_x$, we obtain:

$$N = \frac{-A P}{m\left(\frac{\Delta v_x}{\Delta t}\right)} = \frac{A P \Delta t}{2 m v_x}$$

b) Eq. 1 - 15 shows how to derive the equation :

$$m v_x^2 = k T \Rightarrow v_x = \sqrt{k T / m}$$

c) In part a), we calculated the number N of particles striking an area A on the wall of a cylinder. If there were a hole of area A, then this number of particles would exit the cylinder each second. Therefore, the rate of decrease of particles from the cylinder, dN/dt is equal to the number of particles striking this area each second. Using the results from part a), this gives us :

$$\frac{dN}{dt} = - \frac{A P}{2 m v_x}$$

(the minus sign indicates that dN/dt is negative). Use the ideal gas law to set P = N k T/V and use the result of part b) to express v_x; this gives us:

$$\frac{dN}{dt} = - \frac{A (N k T / V)}{2 m \sqrt{k T / m}} = \frac{A}{2 V} \sqrt{k T / m} \; N$$

for a system of fixed size at a constant temperature, the only variable on the right hand side is N, so we can define the constant :

$$\tau = \left(\frac{A}{2 V} \sqrt{k T / m} \right)^{-1}$$

where τ is the e - folding or characteristic time for this system. With this definition, our differential equation simplifies to :

$$\frac{dN}{dt} = -N / \tau \Rightarrow \frac{dN}{N} = - dt / \tau$$

This is an almost trivial first order differential equation, whose solution is :

$$N(t) = N(0) e^{-t/\tau}$$

f) This problem essentially asks us to find the characteristic time of effusion for the Verne spaceship. Let' s say there is a window of area 1 m^2(a circular window of radius 56 cm seems reasonable) . For a crew of say three people, a volume of 125 m^3(a 5m x 5m x 5m space) seems about right (about the size of a large bedroom with a really tall ceiling). Assume room temp of approximately 300K and an atmosphere of oxygen and we have:

$$\tau = \left(\frac{1 \, m^2}{2 \, x \, 125 \, m^3} \sqrt{\frac{1.38 \times 10^{-23} \, J / K \cdot 300 \, K}{32 \cdot 1.6 \times 10^{-27} \, kg}} \right)^{-1} = 0.88 \, s$$

This means that the atmospheric pressure inside the capsule will decrease by a factor of 1/e (or a loss of 63 % of the atmosphere) in 0.88 s. It seems unlikely that they could open and close a window in much less than that time, so they should think of another means of dead dog disposal.

---

3. Let' s assume a 200 g (0.2 kg) cup of water (as suggested in problem 1.29) and an initial tempera-

ture of 20 C. The time it will take to warm the water to 100 C is :

$$\text{time} = \frac{\text{energy needed to heat 0.2 kg from 20 C to 100 C}}{\text{power output of microwave}} = \frac{0.2\text{ kg} \cdot 4200\text{ J/kg/C} \cdot 80\text{ C}}{600\text{ J/s}} = 112\text{ s}$$

In the last equation, I approximated the specific heat of water as 4200 J/kg/C . There is no heat in this case since the energy transferred to the water was via work, and not because of any temperature difference between objects.

---

4. In each case we use the equation

$$U = \frac{1}{2} N f k T$$

k is Boltzmann' s constant and T is the same in both cases (room temp), so these values do not need to be changed. If we equate the ideal gas law with equipartition we find :

$$PV = N k T \Rightarrow U = \frac{1}{2} f P V$$

Since P and V are the same in both cases, we need only change the degrees of freedom : f will be 3 for the monatomic gas (Helium), and 5 for the diatomic gases (oxygen and nitrogen) comprising air. This difference arises from the two rotational degrees of freedom for diatomics. While I did not ask you to calculate actual values of thermal energy, it is easy to do so. For P=1 atm and V= 1L, use the conversions:

$$1\text{ atm} = 10^5\text{ N}/\text{m}^2 \text{ and } 1\text{ L} = 10^{-3}\text{ m}^3$$

we have that PV for these paramters = 100 Nm = 100 J, so that f PV for the monatomic gas is 150 J and for the diatomic gas is 250 J.

---

5. a) Your graph should plot P (atm) vs. V (L); the curve will be a straight line from (1, 1) to (3, 3). The slope of the line is 1 atm/L.

b) Since W - $\int$P dV, you could find the work by simply calculating the area under the line in part a) using elementary geometry, or you could write the equation of the line in a) as:

$$P(V) = 1\,(\text{atm}/\text{L})\,V$$

so that :

$$W = -\int_{V_i}^{V_f} P(V)\,dV = -\int_{1\text{ L}}^{3\text{ L}} 1\,(\text{atm}/\text{L})\,V\,dV = -4\text{ lit}\cdot\text{atm}$$

From question 4, we know that 1 lit·atm = 100 J, so the work done in this process is - 400 J. The negative sign means that work is being done by the gas on its environment.

c) The thermal energy is given by U = f N k T/2. For a monatomic gas, f = 3 and U = (3/2) N k T = (3/2) P V (making use of the ideal gas law to rewrite N k T). Thus, the change in U is :

$$\Delta U = \frac{3}{2}[P_f V_f - P_i V_i] = \frac{3}{2}[9\,\text{lit}\cdot\text{atm} - 1\,\text{lit}\cdot\text{atm}] = 12\,\text{lit}\cdot\,\text{atm} = 1200\,\text{J}$$

d) By the First Law :

$$\Delta U = W + Q \Rightarrow Q = \Delta U - W = +1600\,\text{J}$$

e) In this process, heat flows into the system, causing the gas to expand and do work on the environment. This process could occur if a source of energy, say a flame or other heating element heated the gas causing it to expand in this linear way.