# PHYS 380

# HOMEWORK #1--SOLUTIONS

1. Consider a standard 12 hour clock with both an hour and a minute hand. The two hands are aligned at noon. What is the next time the two hands are aligned? How does this question relate to anything discussed in Chapter 1?

This problem is essentially asking to find the synodic period of the hour hand with respect to the minute hand. If you consider the set of equations presented in (1.1) on p. 6 of the text, you will see that they are written in the form :

$$\frac{1}{S} = \frac{1}{P_{\text{faster}}} - \frac{1}{P_{\text{slower}}}$$

where S represents the synodic period of the system and the P terms represent the sidereal periods of the faster and slower objects. In this case, the minute hand is the faster object and has a sidereal period of 1 hour, since it takes 1 hour for the minute hand to make a complete revolution around the clock face. The slower hour hand has a sidereal period of 12 hours, implying that the synodic period is :

$$\frac{1}{S} = \frac{1}{1} - \frac{1}{12} = \frac{11}{12}\ \text{hrs} \Rightarrow S = \frac{12}{11}\ \text{hrs} = 1^{h}\,5^{m}\,27.3^{s}$$

You might recognize this question as a staple of high school math competitions.

2. Text, 1.5, page 21. (Btw, the $42^{o}$ parallel runs through Madonna della Strada).

On the first day of summer, the sun is overhead at the Tropic of Cancer at a latitude of $23.5^{o}$N. This implies that Chicago is $42^{o} - 23.5^{o} = 18.5^{o}$ away from this location, further implying that the noon sun at the latitude of Chicago is $18.5^{o}$away from the zenith. In other words, the noon sun on the first day of Chicago summer has an altitude of $90^{0}$ - $18.5^{o}$ = $71.5^{o}$. On the first day of winter, the sun is overhead at the Tropic of Capricorn whose latitude is $-23.5^{o}$. Thus the noon sun on the first day of Chicago winter is $90 - (42 - (-23.5)) = 24.5^{o}$ above the horizon.

3. Calculate these values for a location at the arctic circle and for a location at the tropic of Cancer.

The latitude of the Arctic Circle is $66.5^0$, so the noon sun angle on the first day of summer is $90 - (66.5 - 23.5) = 23.5^o$. On the first day of winter, the noon sun angle is 90-(66.5-(-23.5))=0, meaning the sun never rises on the first day of winter anywhere at or above 66.5N.

At the tropic of Cancer, the sun is directly overhead on the first day of the summer, so the noon sun angle is $90^o$. On the first day of winter, the noon sun angle is $90 - (23.5 - (-23.5)) = 43^o$, approximately the noon sun angle that occurs in Chicago on March 7 or October 5.

4. Text, 1.10, page 22. You may use the celestial spheres to help you with this question

In January, the right ascension of the sun is approximately 19 hours, so that the stars opposite the sun (and thus most visible during the long northern hemisphere winter nights) are those 12 hours distant from the sun, or stars with RAs of 7 hrs plus or mins a few hours.

5. Text, 1.8, page 22. Remember that right ascension is given in time, and 1 hour of right ascension = 15 degrees of arc.

If the two points in the sky are sufficiently close to each other (as they are in this case), we can use the Pythagorean theorem in the form of equation (1 - 8) from p. 19 of the text :

$$(\Delta\theta)^2 = (\Delta\alpha \cos\delta)^2 + (\Delta\delta)^2$$

where $\theta$ is the angle between the two objects on the celestial sphere, $\Delta\alpha$ is the difference in right ascension and $\Delta\delta$ is the difference in declination between the two objects.

We first find the difference in RA and dec for the two objects using the data provided:

$$\Delta\alpha = 14^{\mathrm{h}}\,29^{\mathrm{m}}\,42.95^{\mathrm{s}} - 14^{\mathrm{h}}\,39^{\mathrm{m}}\,36.5^{\mathrm{s}} = 9^{\mathrm{m}}\,53.55^{\mathrm{s}} = 9.893^{\mathrm{m}}$$
$$\Delta\delta = -62^{\mathrm{o}}\,40^{'}\,46.1^{''} - \left(-60^{\mathrm{o}}\,50^{'}\,02.3^{''}\right) = 1^{\mathrm{o}}\,50^{'}\,43.8^{''}$$

Since we only care about magnitudes of differences, we can neglect the negative signs above. Now, recall that RA is measured in hours, so that 9.893 minutes of RA = 0.165 hours of RA. Since each hour of RA = 15 degrees of arc, the computed difference in RA is equal to an angular separation of 0.165 hours * 15 deg/hour = 2.473 degrees. Substituting into equation (1 - 8), we get :

$$(\Delta\theta)^2 = (\Delta\alpha \cos\delta)^2 + (\Delta\delta)^2 = (2.473 \cos 62.679)^2 + (1.8455)^2$$
$$\Delta\theta = 2.17^{o}$$

If the distance from the Earth to Proxima Centauri is r and the distance between the two stars is d, simple trig allows us to write that

$$\tan\theta = \frac{d}{r} \Rightarrow d = r\tan\theta = 4\times 10^{16} \tan 2.17 = 1.51\times 10^{15} \text{ m.}$$

To put this is context, 1 ly (light year) = 9.45 $10^{15}$ m, so this represents approximately 0.16 ly. Pluto is, on average, 5 light ***hours*** from the sun, so you can appreciate that an ongoing question in astronomy has been whether Proxima Centauri is gravitationally bound to $\alpha$ Cen or whether their proximity in space is just an optical coincidence. (It appears that they are in fact a bound system.)