PHYS 380

HOMEWORK #5 -- SOLUTIONS

1. 7.2 from text : We want to find the average value of $\sin^3 i$ from 0 to 2 π using a weighting function. We know that the probability of observing radial velocities is zero when i = 0, and greatest when i = 90. This strongly suggests that the weighting funciton should be w (i) = sin i. Therefore,

$$< \sin^3 i > = \int_0^{\pi/2} \sin i * \sin^3 i \; di$$

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In[12]:= Integrate[Sin[i]^4, {i, 0, π / 2}]

Out[12]= 3 π
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          16
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2. 7.6 from text :

a) We know that the ratio of masses is related to the inverse ratio of radial velocities :

$$\frac{m_B}{m_A} = \frac{v_{rA}}{v_{rB}} = \frac{5.4}{22.4} = 0.241$$

b) Assuming the inclination is close to 90 degrees (which is very reasonable for an eclipsing binary), we have :

$$m_A + m_B = \frac{P}{2\pi G}\frac{(v_{Ar} + v_{Br})^3}{\sin^3 i} = \frac{6.31 \text{ yr} * 3.15 \times 10^7 \frac{\text{s}}{\text{yr}}}{2\pi * 6.67 \text{ x}10^{-11}} * \frac{\left(5400 \frac{\text{m}}{\text{s}} + 22\,400 \frac{\text{m}}{\text{s}}\right)^3}{\sin^3 90} =$$
$$1.01 \text{ x}10^{31} \text{ kg} = 5.1 \text{ solar masses}$$

c) Combine the results of a) and b) to find that

$$m_B = 0.241\, m_A \Rightarrow 1.241\, m_A = 5.1 \text{ solar masses} \Rightarrow m_A = 4.1 \text{ solar masses},$$
$$m_B = 1 \text{ solar mass}$$

d) We use the timing data to find :

$$r_{smaller} = \frac{(v_A + v_B)}{2}(t_b - t_a) =$$

$$\left(\frac{5400\,m/s + 22\,400/s}{2}\right)(0.58\,d * 86\,400\,s/d) = 6.97\,x10^8\,m = 1\,\text{solar radius}$$

$$r_{larger} = r_{smaller} + \frac{(v_A + v_B)}{2}(t_c - t_b) = 1\,R_{sun} + \left(\frac{0.64}{0.58}\,R_{sun}\right) = 2.11\,R_{sun}$$

e) Using the definitions in the text, and using definitions of magnitude and flux :

$$\frac{B_\rho}{B_0} = 100^{(5.4-9.2)/5} = 0.0302 \qquad \frac{B_s}{B_0} = 100^{(5.40-5.44)/5} = 0.964$$

and the ratio of temperatures from :

$$\frac{T_s}{T_l} = \left(\frac{\left(1 - \frac{B_\rho}{B_\square}\right)}{\left(1 - \frac{B_s}{B_0}\right)}\right)^{1/4} = 2.28$$

7.10 Larger planets (Jupiter like) will exert a greater gravitational pull on its central star thus generating larger radial velocities. If the planet is close to the star (like 51 Peg), there will be a short orbital period and at least one (and perhaps many) revolutions can be observed in a few years of observation.

7.13 If we ignore the radiation from Jupiter, we want to determine the ratio of light occulted by Jupiter. This ratio is equal to the ratio of Jupiter' s surface area to the sun' s surface area, which in turn is equal to the ratio of radii squared :

$$\frac{F_{eclipse}}{F_{normal}} = \left(\frac{R_{Jupiter}}{R_{Srun}}\right)^2 = \left(\frac{6.99\,x10^7\,m}{6.96\,x10^8\,m}\right)^2 = 0.01$$

8.4 Start with the Maxwell probability distribution :

$$n_v\,dv = n\left(\frac{m}{2\,\pi\,k\,T}\right)^{3/2} e^{\frac{-m\,v^2}{2\,k\,T}}\,4\,\pi\,v^2\,dv$$

We find the most probable velocity by differentiating n_v dv and setting it equal to zero:

$$\frac{d_{n_v}}{dv} = 0 \Rightarrow n\left(\frac{m}{2\pi k T}\right)^{3/2} \frac{d}{dv}\left[e^{\frac{-m v^2}{2 k T}} 4\pi v^2\right] =$$

$$4\pi n\left(\frac{m}{2\pi k T}\right)^{3/2}\left[-\frac{m v}{k T} e^{\frac{-m v^2}{2 k T}} \cdot v^2 + 2 v e^{\frac{-m v^2}{2 k T}}\right] = 0$$

This yields :

$$\frac{m v^3}{k T} = 2 v \Rightarrow v = \sqrt{\frac{2 k T}{m}}$$

8.5 We want to find the ratio of H atoms in n = 2 to H atoms in n = 1. We use the Boltzmann equation :

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \frac{e^{-E_2/k T}}{e^{-E_1/k T}}$$

As discussed in class, the statistical weight of each energy level is $2 n^2$ so g_1=2 and g_2 =8. The ground state energy for H is -13.6 eV, and the energy of the first excited state is E_2 =-3.4 eV, so that the energy difference between states is 10.2 eV. Substituting these into the Boltzmann equation we obtain:

$$\frac{N_2}{N_1} = \frac{8}{2} \operatorname{Exp}\left[\frac{(-10.2 \text{ eV} * 1.6 \times 10^{-19} \text{ J/eV})}{1.38 \times 10^{-23 \text{ J/K}} \text{ T}}\right] = 4 \operatorname{Exp}[-118\,261 / \text{T}]$$

We find the temperature where the ratio of atoms in n = 2 is 0.01 of the atoms in n = 1 :

$$0.01 = 4 \operatorname{Exp}[-118\,261 / \text{T}] \Rightarrow -118\,261 / \ln\left(\frac{0.01}{4}\right) = \text{T} = 19\,738 \text{ K}$$

The temperature at which 10 % of the atoms are in n = 2 is

$$\text{T} = -118\,261 / \ln\left(\frac{0.1}{4}\right) = 32\,059 \text{ K}$$