

PHYS 380

HOMEWORK #8--Solutions

1. Text 10.3, p. 345.

For a star of one solar mass to be composed purely of hydrogen, the total number of atoms is

$$\text{number atoms} = \frac{\text{mass of sun}}{\text{mass / H atom}} = \frac{2 \times 10^{30} \text{ kg}}{1.6 \times 10^{27} \text{ kg / atom}} = 1.2 \times 10^{57} \text{ atoms}$$

If each atom releases 10 eV of energy per reaction, then the sun could produce energy at its current rate for a time of :

$$\text{time} = 10 \text{ eV / atom} \times 1.2 \times 10^{57} \text{ atoms} / 3.84 \times 10^{26} \text{ J / s}$$

Converting eV to J :

$$\text{time} = \frac{1.6 \times 10^{-18} \text{ J / atom} \times 1.2 \times 10^{57} \text{ atoms}}{3.84 \times 10^{26} \text{ J / s}} = 5 \times 10^{12} \text{ s} = 1.59 \times 10^5 \text{ yr}$$

2. Text 10.4, p. 345.

a) We find the rms velocity by equating kinetic energy to thermal energy :

$$\frac{1}{2} \mu v_{\text{rms}}^2 = \frac{3}{2} k T$$

where μ is the reduced mass of a proton-proton system. If we assume that $v \geq 10 v_{\text{rms}}$, then we have

$$v = 10 v_{\text{rms}} = 10 (3 k T / \mu)^{1/2}$$

Additionally, in the classical limit, the kinetic energy must exceed the Coulombic potential barrier :

$$\frac{1}{2} \mu v^2 = \frac{k_c e^2}{r}$$

where e is the charge of a proton, r is the separation of protons (assume 1 fm), and k_c is the constant equal to $1/4 \pi \epsilon_0$, we have:

$$\frac{k_c e^2}{r} = \frac{1}{2} \mu v^2 = \frac{1}{2} \mu \left(10 \sqrt{3 k T / \mu} \right)^2 = 50 (3 k T)$$

Solving for T :

$$T = \frac{k_c e^2}{150 k r} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19} \text{ C})^2}{150 \times 1.38 \times 10^{-23} \text{ J/K} \times 1 \times 10^{-15} \text{ m}} = 1.1 \times 10^8 \text{ K}$$

b) We will use the Maxwellian velocity distribution; taking ratios only of the terms involving v (omitting constants and non velocity related factors since they will cancel out in the ratio) we get :

$$\frac{n_v}{n_{\text{rms}}} = \frac{e^{-m (10 v_{\text{rms}})^2 / 2 k T} (10 v_{\text{rms}})^2}{e^{-m v_{\text{rms}}^2 / 2 k T} v_{\text{rms}}^2} = 100 e^{-99 m v_{\text{rms}}^2 / 2 k T} = 100 e^{-99 (m/2 k T) (3 k T / m)}$$

where n_v is the number of particles with velocities in the range of $v + dv$, and n_{rms} is the number of particles with velocities in the range $v_{\text{rms}} + dv$. In the last step in the exponential above, we replace v_{rms}^2 with the expression $v_{\text{rms}}^2 = 3 k T / m$, and we get:

$$\frac{n_v}{n_{\text{rms}}} = 100 e^{-99 (1.5)} = 3.2 \times 10^{-63}.$$

This seems low, but it is what several of us have gotten; this makes part c) easy : no, there are essentially no atoms fast enough to generate energy according to this criteria. The moral of the story : quantum tunneling is a good thing.

3. Text 10.7, p. 345.

We proved earlier in the course that the gravitational binding energy of a self - gravitating sphere is :

$$U = \frac{-3}{5} \frac{G M^2}{R}$$

using solar values, we obtain :

$$U = \frac{-3}{5} \frac{(6.67 \times 10^{-11} \text{ MKS})(2 \times 10^{30} \text{ kg})^2}{7 \times 10^8 \text{ m}} = -2.29 \times 10^{41} \text{ J}$$

Using the virial theorem, we know that

$$2 T + V = 0 \Rightarrow V = -2 T$$

We know that the temperature of a gas is related to its kinetic energy through :

$$\frac{1}{2} m v^2 = \frac{3}{2} N k T$$

where N is the number of particles and k is the Boltzmann constant. Therefore,

$$2 T = m v^2 = 3 N k T = 2.3 \times 10^{41} \text{ J}$$

We showed earlier in this set that there are approximately 10^{57} atoms in the sun; if we assume 100% ionization of hydrogen, then there are 2×10^{57} particles in the sun, and we estimate the temperature from:

$$T = \frac{2.3 \times 10^{41} \text{ J}}{3 (2 \times 10^{57} \text{ particles}) (1.38 \times 10^{-23} \text{ J/K})} = 2.8 \times 10^6 \text{ K}$$

which is low for the center of the sun, and is approximately equal to the solar temperature at a radius of $0.6 R_{\text{sun}}$ (according to the models presented in Ch. 11).

4. Text 10.12, p. 346

Using the masses given in the text and the steps in the PP I chain, as well as the conversion that 1 atomic mass unit = $931.494 / c^2$ (see table of physical constants, inside cover of text) :

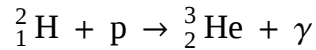
Step 1 :

$$p + p \rightarrow {}^2_1\text{H} + e^+ + \nu_e$$

$$Q_1 = (2 m_p - m_{{}^2_1\text{H}} - m_e) c^2 =$$

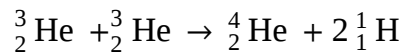
$$(2 * 1.0078 \text{ u} - 2.0141 \text{ u} - 5.3486 \times 10^{-4} \text{ u}) c^2 = 0.9 \text{ MeV}$$

Step 2 :



$$Q_2 = (m_{{}^2_1\text{H}} + m_p - m_{{}^3_2\text{He}}) c^2 = (2.0141 \text{ u} + 1.0078 \text{ u} - 3.016 \text{ u}) c^2 = 5.5 \text{ MeV}$$

Step 3 :



$$Q_3 = (2 m_{{}^3_2\text{He}} - m_{{}^4_2\text{He}} - 2 m_p) c^2 = (2 * 3.016 \text{ u} - 4.0026 \text{ u} - 2 * 1.0078 \text{ u}) c^2 =$$

$$12.87 \text{ MeV}$$

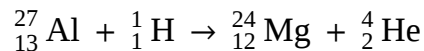
5. Text 10.14, p. 346.

a)



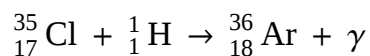
The neutrino must be emitted to conserve lepton number, and we know the atomic mass of Al must equal the atomic mass of Si to conserve baryon number.

b)



This reaction is easy to balance, keeping in mind conservation of charge and baryon number.

c)



The easiest of all; baryon number and charge are conserved, the photon carries away energy as required by conservation of momentum and matter - energy. (Why don't we have a positron /electron neutrino pair on the right?)