PHYS 380
HOMEWORK #8--Solutions

1. Text 10.3, p. 345.

For a star of one solar mass to be composed purely of hydrogen, the total number of atoms is

$$\text{number atoms} = \frac{\text{mass of sun}}{\text{mass / H atom}} = \frac{2 \text{ x } 10^{30} \text{ kg}}{1.6 \text{ x} 10^{27} \text{ kg / atom}} = 1.2 \text{ x} 10^{57} \text{ atoms}$$

If each atom releases 10 eV of energy per reaction, then the sun could produce energy at its current rate for a time of :

$$\text{time} = 10 \text{ eV / atom x } 1.2 \text{ x} 10^{57} \text{ atoms / } 3.84 \text{ x} 10^{26} \text{ J / s}$$

Converting eV to J :

$$\text{time} =$$
$$1.6 \text{ x} 10^{-18} \text{ J / atom x } 1.2 \text{ x } 10^{57} \text{ atoms / } 3.84 \text{ x} 10^{26} \text{ J / s} = 5 \text{ x} 10^{12} \text{ s} = 1.59 \text{ x} 10^{5} \text{ yr}$$

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2. Text 10.4, p. 345.

a) We find the rms velocity by equating kinetic energy to thermal energy :

$$\frac{1}{2} \mu \text{ v}^2{}_{\text{rms}} = \frac{3}{2} \text{ k T}$$

where μ is the reduced mass of a proton-proton system. If we assume that v $\geq 10 \; v_{\text{rms}}$, then we have

$$\text{v} = 10 \text{ v}_{\text{rms}} = 10 \, (3 \text{ k T} / \mu)^{1/2}$$

Additionally, in the classical limit, the kinetic energy must exceed the Coulombic potential barrier :

$$\frac{1}{2} \mu \text{ v}^2 = \frac{\text{k}_\text{c} \text{ e}^2}{\text{r}}$$

where e is the charge of a proton, r is the separation of protons (assume 1 fm), and k_c is the constant equal to $1/4\,\pi\,\epsilon_0$, we have:

$$\frac{k_c\,e^2}{r} = \frac{1}{2}\,\mu\,v^2 = \frac{1}{2}\,\mu\left(10\sqrt{3\,k\,T/\mu}\right)^2 = 50\,(3\,k\,T)$$

Solving for T :

$$T = \frac{k_c\,e^2}{150\,kr} = \frac{9\,x10^9\,x\left(1.6\,x10^{-19}\,C\right)^2}{150\,x\,1.38\,x10^{-23}\,J/K\,x\,1\,x10^{-15}\,m} = 1.1\,x10^8\,K$$

b) We will use the Maxwellian velocity distribution; taking ratios only of the terms involving v (omitting constants and non velocity related factors since they will cancel out in the ratio) we get :

$$\frac{n_v}{n_{rms}} = \frac{e^{-m\,(10\,v_{rms})^2/2\,k\,T}\,(10\,v_{rms})^2}{e^{-m\,v_{rms}{}^2/2\,k\,T}\,v_{rms}{}^2} = 100\,e^{-99\,mv_{rmw}{}^2/2\,k\,T} = 100\,e^{-99\,(m/2\,kT)\,(3\,kT/m)}$$

where n_v is the number of particles with velocities in the range of v + dv, and $n_{\rm rms}$ is the number of particles with velocities in the range $v_{\rm rms}$ +dv. In the last step in the exponential above, we replace $v_{\rm rms}^2$ with the expression $v_{\rm rms}^2$ = 3 k T/m, and we get:

$$\frac{n_v}{n_{rms}} = 100\,e^{-99\,(1.5)} = 3.2\,x\,10^{-63}.$$

This seems low, but it is what several of us have gotten; this makes part c) easy : no, there are essentially no atoms fast enough to generate energy according to this criteria. The moral of the story : quantum tunneling is a good thing.

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3. Text 10.7, p. 345.

We proved earlier in the course that the gravitational binding energy of a self - gravitating sphere is :

$$U = \frac{-3}{5}\frac{G M^2}{R}$$

using solar values, we obtain :

$$U = \frac{-3}{5}\frac{\left(6.67\text{ x }10^{-11}\text{ MKS}\right)\left(2\text{ x }10^{30}\text{ kg}\right)^2}{7\text{ x}10^8\text{ m}} = -2.29\text{ x}10^{41}\text{ J}$$

Using the virial thereom, we know that

$$2\,T + V = 0 \Rightarrow V = -2\,T$$

We know that the temperature of a gas is related to its kinetic energy through :

$$\frac{1}{2}\,m\,v^2 = \frac{3}{2}\,N\,k\,T$$

where N is the number of particles and k is the Boltzmann constant. Therefore,

$$2\,T = m\,v^2 = 3\,N\,k\,T = 2.3\text{ x }10^{41}\text{ J}$$

We showed earlier in this set that there are approximately 10^{57} atoms in the sun; if we assume 100% ionization of hydrogen, then there are $2\text{ x}10^{57}$ particles in the sun, and we estimate the temperature from:

$$T = \frac{2.3\text{ x }10^{41}\text{ J}}{3\left(2\text{ x}10^{57}\text{ particles}\right)\left(1.38\text{ x}10^{-23}\text{ J}/\text{K}\right)} = 2.8\text{ x}10^6\text{ K}$$

which is low for the center of the sun, and is approximately equal to the solar temperature at a radius of 0.6 R_{sun} (according to the models presented in Ch. 11).

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4. Text 10.12, p. 346

Using the masses given in the text and the steps in the PP I chain, as well as the conversion that 1 atomic mass unit = $931.494 \big/ c^2$ (see table of physical constants, inside cover of text) :

Step 1 :

$$p + p \to {}^{2}_{1}H + e^{+} + \nu_e$$

$$Q_1 = \left(2\, m_p - m_{\,^2H} - m_e\right) c^2 =$$

$$\left(2 * 1.0078\, u - 2.0141\, u - 5.3486\, x10^{-4}\, u\right) c^2 = 0.9\, \text{MeV}$$

Step 2 :

$${}^{2}_{1}H + p \to {}^{3}_{2}He + \gamma$$

$$Q_2 = (m_{2\,H} + m_p - m_{He3})\, c^2 = (2.0141\, u + 1.0078\, u - 3.016\, u)\, c^2 = 5.5\, \text{MeV}$$

Step 3 :

$${}^{3}_{2}He + {}^{3}_{2}He \to {}^{4}_{2}He + 2\,{}^{1}_{1}H$$

$$Q_3 = (2\, m_{He3} - m_{He4} - 2\, m_p)\, c^2 = (2 * 3.016\, u - 4.0026\, u - 2 * 1.0078\, u)\, c^2 =$$

$$12.87\, \text{MeV}$$

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5. Text 10.14, p. 346.

a)

$${}^{27}_{14}Si \to {}^{27}_{13}Al + e^{+} + \nu_e$$

The neutrino must be emitted to conserve lepton number, and we know the atomic mass of Al must equal the atomic mass of Si to conserve baryon number.

b)

$${}^{27}_{13}Al + {}^{1}_{1}H \to {}^{24}_{12}Mg + {}^{4}_{2}He$$

This reaction is easy to balance, keeping in mind conservation of charge and baryon number.

c)

$${}^{35}_{17}Cl + {}^{1}_{1}H \to {}^{36}_{18}Ar + \gamma$$

The easiest of all; baryon number and charge are conserved, the photon carries away energy as required by conservation of momentum and matter - energy. (Why don' t we have a positron /electron neutrino pair on the right?)