KEPLER'S LAWS OF PLANETARY MOTION

In the early 1600s, Johannes Kepler culminated his analysis of the extensive data taken by Tycho Brahe and published his three laws of planetary motion, which we know today as ***Kepler's Laws.***

Kepler's First Law describes the nature of planetary orbits. This law describes that planets revolve around the sun in **elliptical** orbits, and that the sun is at one of the *foci* of the ellipse. Because planets do not travel in perfectly circular orbits, the distance between the planet and the sun varies throughout the course of the year. For the Earth, this annual distance variation amounts to only 3% and importantly is **not the cause of seasonal weather variations on the earth.** Please refer to our companion page http://www.luc.edu/faculty/dslavsk/classnotes/phys478/reasons-for-seasons.pdf for more details on the Earth's orbit and the reasons for seasons. At the end of this write-up I show plots of several ellipses; all the ellipses have a semi-major axis of 2, and the eccentricity varies from 0 to 0.9 so you can see how eccentricity effects the shape of an ellipse.

The diagram below will help illustrate the meaning and importance of ***Kepler's Second Law***.

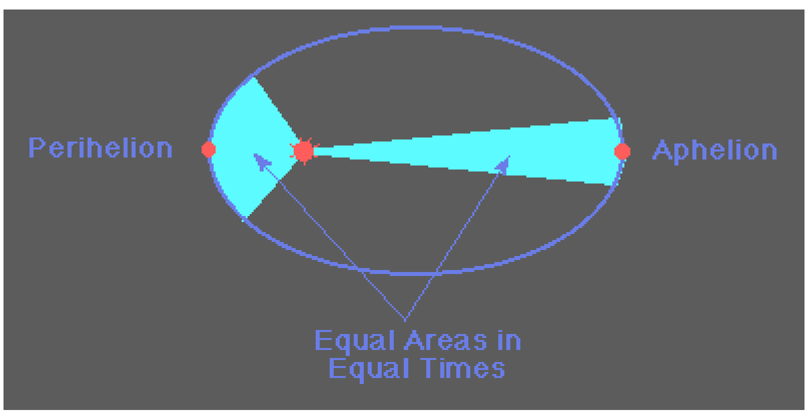


Image courtesy Department of Physics and Astronomy, University of Tennessee at Knoxville.

The diagram shows a planet orbiting the sun in a clearly elliptical orbit. The planet is shown at two separate times during its orbit; the **perihelion** being the closest point in the orbit to the sun, and the **aphelion** being the most distant point in the orbit from the sun. For the Earth, perihelion occurs around January 3 of each year and aphelion occurs around July 4 and is a holiday exuberantly celebrated in the United States.

Refer again to the diagram above. The area "swept out" by a planet is determined by drawing a straight line from the planet to the sun at the beginning of the time interval and also at the end of the interval. When the planet is near perihelion, it is closest to the sun and thus experiences a greater gravitational force. This means the planet moves faster (since it undergoes more acceleration in response to the greater force) and thus travels a greater arc around the sun. At aphelion, the planet does not travel through such a great arc since it travels more slowly in response to the diminished gravitational force.

Now if we compare the motion of the planet for one month when it is near perihelion to the motion of the planet for one month when it is near aphelion, we will see that the ***area*** swept out by the planet is exactly the same. Since the area depends on both the distance from the sun and the arc length traveled by the planet, Kepler showed that the decrease in arc length is exactly compensated by the increase in distance from the sun, so that the area. Kepler's Second Law is often stated as: **The line joining the planet to the Sun sweeps out equal areas in equal times as the planet travels around the ellipse.**

When Kepler published his work, he did not have any theoretical model to explain their existence; he knew only that they were consistent with the data Tycho Brahe had amassed over twnenty years of observing.

It was not until Newton provided the basis of modern physics that Kepler's Laws could be understood in terms of gravitational forces. It can be shown in more advanced physics courses that Kepler's Second Law is a result of the conservation of angular momentum of the planet orbiting the sun.

Published in 1619 in his *Harmonices Mundi*¸ the **Third Law** of Planetary Motion relates the period of a planet's orbit to the size of the semi-major axis of its orbit. Recall that the period (P) of a planet's orbit is the time required for one complete orbit around the sun, and that the semi-major axis (d) of a planet's orbit is its average distance from the sun.

Kepler found a simple relationship between these two important orbital parameters. This relationship is especially simple if we use astronomically based units to measure period and semi-major axis.

In this system of units, we measure distances (semi-major axes) in Astronomical Units (AU) and we measure time (periods) in years. One AU is the average Earth-sun distance, and of course one year is the period of the Earth's orbit around the sun.

Thus, for orbits around the sun, Kepler's Third Laws tells us that the period and semi-major axis of a planetary orbit are related by the following simple expression:

$$P^2 = d^3$$

Let's verify that this equation holds for the Earth. The period of the Earth's orbit is one year, and the semi-major axis of the orbit is 1 AU. These data imply that Kepler's Third Law becomes:

$$P^2 = d^3 \Rightarrow 1^2 = 1^3$$

Since 1^2 does in fact equal 1^3, we see that Kepler's Third Law is valid for the Earth's Orbit.

Suppose we wish to find the period of an asteroid orbiting with a semi-major axis of 4 AU, we would use Kepler's 3rd Law:

$$P^2 = d^3 \Rightarrow P^2 = 4^3 = 64 \text{ so } P = \sqrt{64} = 8\,years$$

So the period of this asteroid is 8 years.

In class we have spent some time talking about the orbit of Pluto. We know that Pluto has a very eccentric orbit ($e = 0.25$) and a semi-major axis of 39.5 AU. What is Pluto's Period around the sun? Again, use Kepler's 3rd Law:

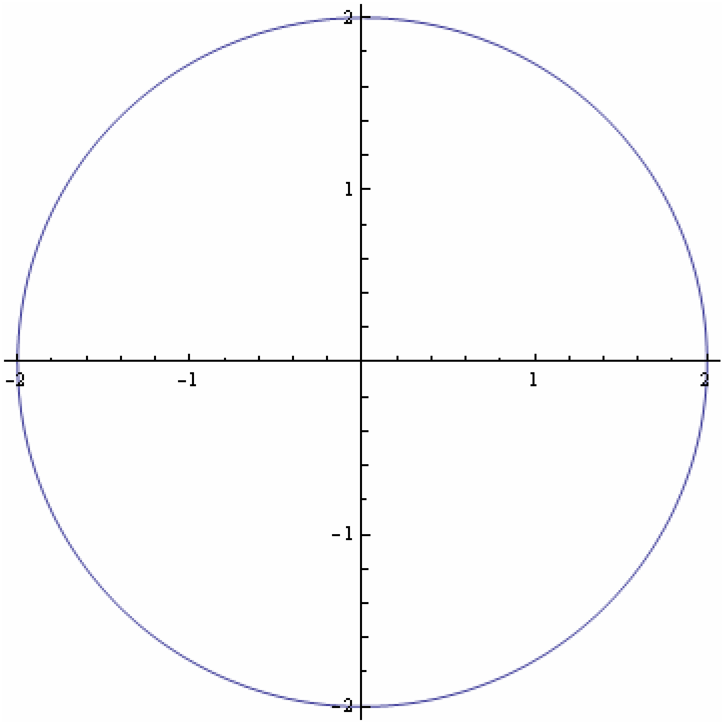
$$P^2 = d^3 \Rightarrow P^2 = 39.5^3 = 61630$$
$$P = \sqrt{61{,}630} = 248\,years$$

Let's try another example. We know that the period of the famous Comet Halley is 76 years; what then is its semi-major axis? Again, we begin with Kepler's third law:

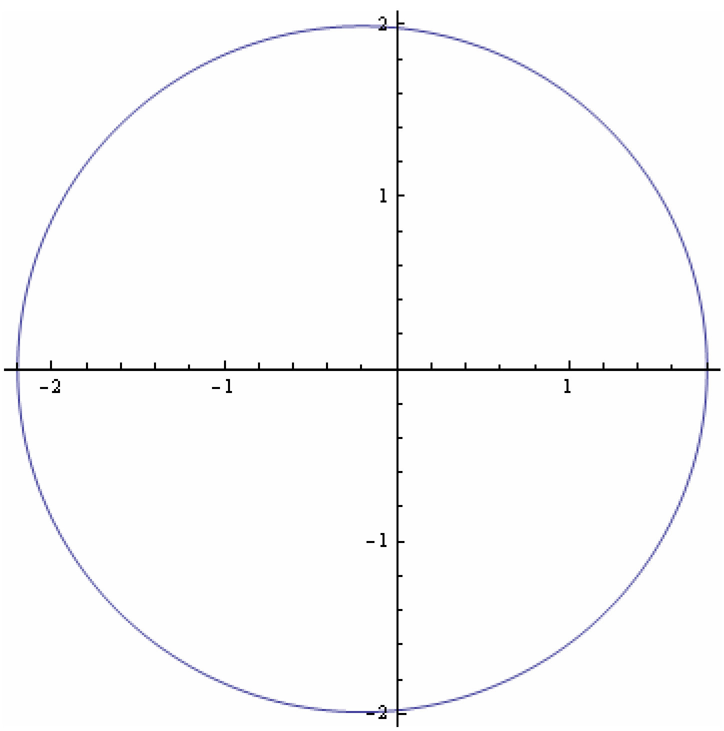
$$P^2 = d^3 \Rightarrow 76^2 = 5776 = d^3$$
$$d = \sqrt[3]{5776} = 17.9AU$$

Remember now that the number we have just computed is the **average** distance between the comet and the sun. Many comets, including Halley have very eccentric elliptical orbits, so that the perihelion and aphelion distances are very different from each other. In the case of Halley, the perihelion distance is less than 1 AU (in fact, about ½ AU), so that the aphelion distance is almost 35.5AU, beyond the orbit of Neptune! This means that Halley spends most of its time in the outer solar system, and passes by the Earth for a very small fraction of its 76 year orbit, making its passage a truly once in a lifetime experience for most people. The last passage of Halley to the inner solar system was in 1985-1986, and the next will be in 2061.

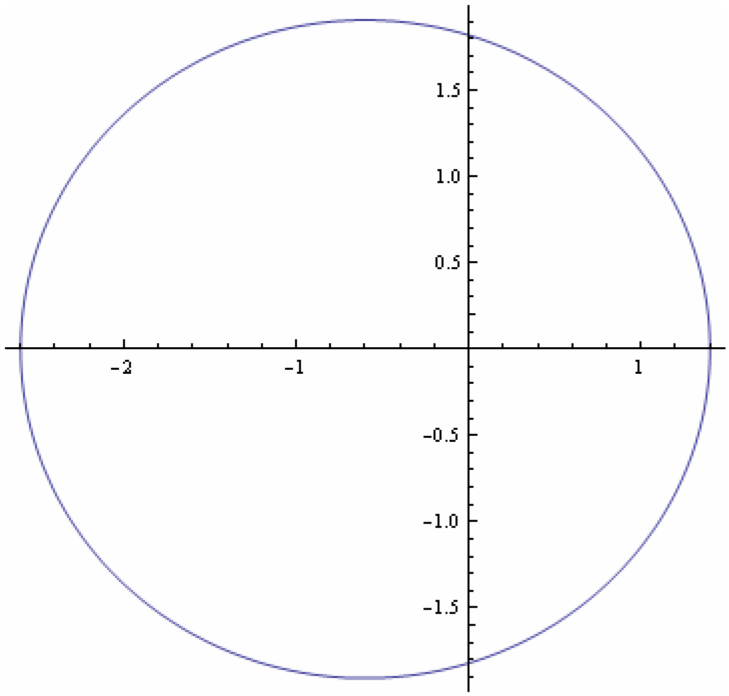
See the pages below for a series of graphs of ellipses of various semi-major axes.



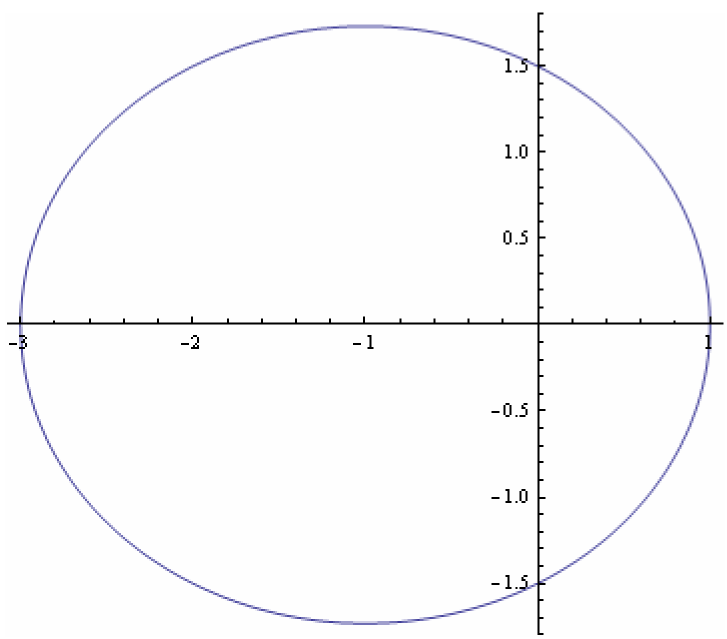
Circle of radius 2 (so the semi-major axis is also 2); all circles have zero eccentricity.



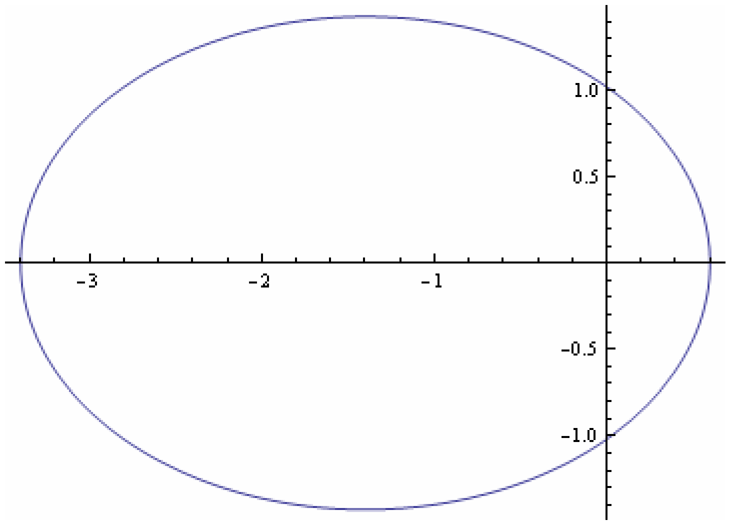
Ellipse with semi-major axis =2 and eccentricity = 0.1



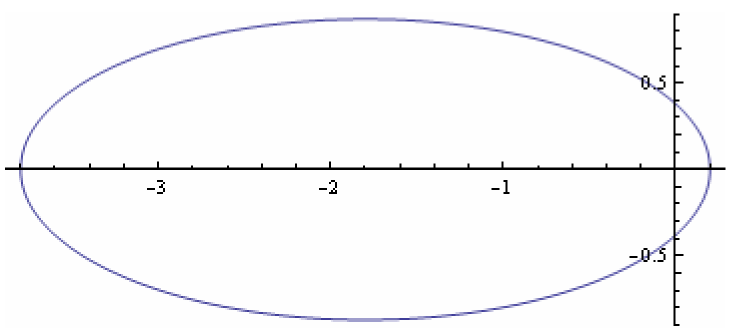
Ellipse; semi-major axis = 2; eccentricity = 0.3



Ellipse; semi-major axis = 2; eccentricity = 0.5



Ellipse; semi-major axis =2; eccentricity = 0.7



Ellipse; semi-major axis = 2 units; eccentricity = 0.9